

# Reflection of light from semi-infinite turbid media

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The simple equation for the spherical albedo of a semi-infinite turbid medium is obtained. The accuracy of the approximation was studied with the numerical solution of the radiative transfer equation. The error is smaller than 10% for water clouds in the visible and the near infrared. © 1998 Optical Society of America [S0740-3232(98)00111-2]

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Studies of light reflection from semi-infinite turbid media are important for many applications, including medical imaging and cloud, snow, and foam optics. The spherical albedo of such media  $R$  can be obtained by integration of the reflection function  $r(\mu, \mu_0, \psi)$  (Ref. 1):

$$R = \frac{2}{\pi} \int_0^1 \mu d\mu \int_0^1 \mu_0 d\mu_0 \int_0^{2\pi} r(\mu, \mu_0, \psi) d\psi, \quad (1)$$

where  $\mu = \cos \nu$ ;  $\mu_0 = \cos \nu_0$ ;  $\nu$  and  $\nu_0$  are the observation and the incidence angles, respectively; and  $\psi$  is the azimuth. The function  $r(\mu, \mu_0, \psi)$  is the solution of the radiative transfer equation.<sup>1</sup> This solution depends on the single scattering albedo  $\omega$  and the phase function  $p(\theta)$ , where  $\theta$  is the scattering angle. The value of  $\omega$  represents the probability that a photon injected into a turbid medium will be scattered after interaction with a particle. The function  $p(\theta)$  is the probability that a photon will scatter in the direction specified by the scattering angle  $\theta$ . According to the similarity principle,<sup>1</sup> turbid media that have different phase functions  $p(\theta)$  but the same value of the asymmetry parameter

$$g = \frac{1}{2} \int_0^\pi p(\theta) \cos \theta \sin \theta d\theta \quad (2)$$

and the single scattering albedo  $\omega$  have approximately the same radiative characteristics. Thus in the first coarse approximation the value of  $R$  depends only on two parameters, namely,  $\omega$  and  $g$ .

An approximate formula for the spherical albedo of semi-infinite turbid media was obtained by Perelman *et al.*<sup>2</sup> within the framework of the path integral formalism:

$$R = \exp \left[ -\alpha \left( \frac{\ln \frac{1}{\omega}}{1-g} \right) \right]^{1/2}, \quad (3)$$

where, according to Perelman *et al.*,<sup>2</sup> the value of  $\alpha$  is a function of  $g$ .

The purpose of this paper is to show that the value of  $\alpha$  should be constant and to compare Eq. (3) with exact radiative transfer calculations.

The following exact asymptotical relation for the spherical albedo of a semi-infinite medium can be obtained from the radiative transfer theory as  $\omega \rightarrow 1$  (Ref. 1):

$$R = 1 - 4 \left[ \frac{1-\omega}{3(1-g)} \right]^{1/2}. \quad (4)$$

This relation is valid for any phase functions and values of  $g$ . It is possible to find the value of  $\alpha$  in Eq. (3) by comparing Eqs. (4) and (3) as  $\omega \rightarrow 1$ . Indeed, it follows from Eq. (3) as  $\omega \rightarrow 1$  that

$$R = 1 - \alpha \left( \frac{1-\omega}{1-g} \right)^{1/2}, \quad (5)$$

where the asymptotical formula

$$\lim_{\omega \rightarrow 1} \left[ \ln \left( \frac{1}{\omega} \right) \right] = 1 - \omega \quad (6)$$

is used. Thus one can see that the value of  $\alpha$  does not depend on  $g$  and that

$$\alpha = \frac{4}{\sqrt{3}}. \quad (7)$$

The value of  $\alpha$  in the paper by Perelman *et al.*<sup>2</sup> was approximately 3.1 at  $g = 0.908$  and approximately 2.9 at  $g = 0.757$ . This value is nearly 30% larger than it should be according to Eq. (7). It follows from Eqs. (3) and (7) that

$$R = \exp \left\{ -4 \left[ \frac{\ln \frac{1}{\omega}}{3(1-g)} \right]^{1/2} \right\}. \quad (8)$$

One can see that

$$\lim_{\omega \rightarrow 0} R(\omega) = 0, \quad \lim_{\omega \rightarrow 1} R(\omega) = 1, \quad (9)$$

as expected. The dependence  $R(\omega)$ , calculated with Eq. (8) at  $g = 0.85$ , is represented in Fig. 1. It is interesting that at  $\omega > 0.9$  ( $R > 0.2$ ) a somewhat simpler formula for calculation of the spherical albedo can be applied [see Eq. (6) and Fig. 1]:

$$R = \exp\left\{-4\left[\frac{1-\omega}{3(1-g)}\right]^{1/2}\right\}. \quad (10)$$

Equation (10) was obtained by Rozenberg<sup>3</sup> for a special case of weakly absorbing media. A comparison of the results of the spherical-albedo calculation obtained with Eq. (8) and with a numerical solution of the radiative transfer equation<sup>1</sup> for semi-infinite water clouds with the gamma particle size distribution

$$f(a) = Ba^6 \exp(-1.5a), \quad (11)$$

where  $B$  is the normalization constant [ $\int_0^\infty f(a) da = 1$ ], is presented in Fig. 2. One can see that the accuracy of the approximate Eq. (8) is high.

In conclusion, a simple approximation (8) was proposed to calculate the spherical albedo of turbid semi-infinite media. This approximation can be used to avoid complex numerical calculations of the total reflectance  $R$  and ab-

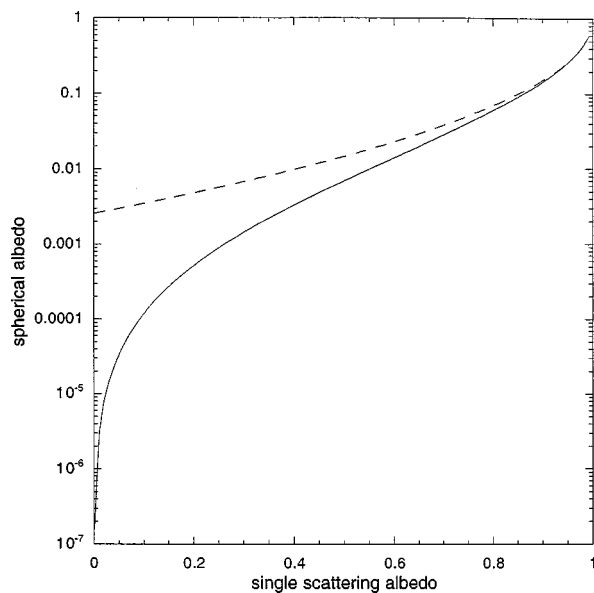


Fig. 1. Dependence of the spherical albedo of a semi-infinite medium on the single scattering albedo at  $g = 0.85$ , calculated with Eq. (8) (solid curve) and with Eq. (10) (dashed curve).

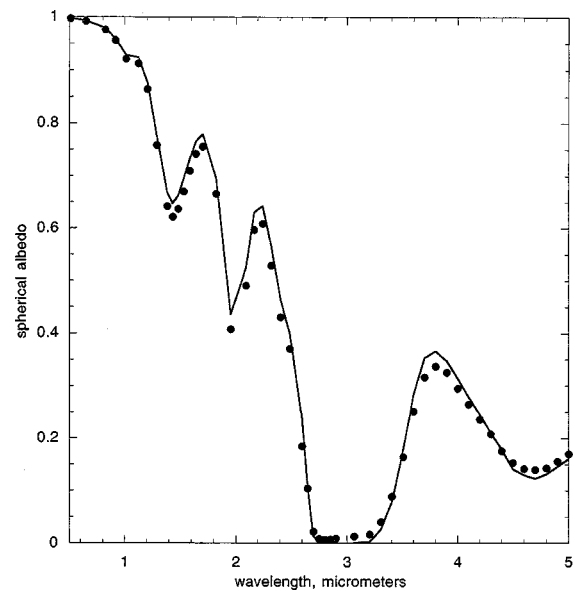


Fig. 2. Dependence of the spherical albedo of a semi-infinite water cloud with particle size distribution (11) on the wavelength, calculated with Eq. (8) (solid curve) and with the numerical solution of the radiative transfer equation (symbols).

sorption  $A = 1 - R$  of radiation by semi-infinite thick turbid media.

## ACKNOWLEDGMENT

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